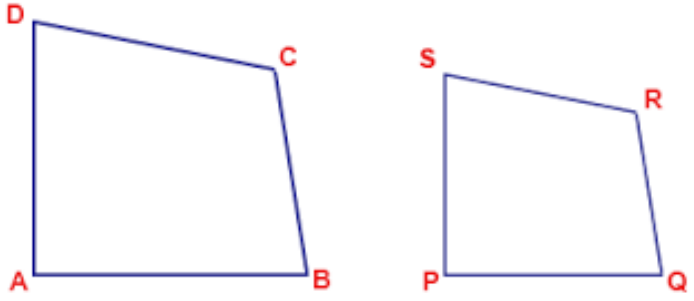
Chapter 6
Similarity

6.1
Use Similar Polygons

Similar Polygons

- Polygons are similar if:
  - Corresponding angles are congruent (≅)
  - AND
  - Corresponding sides are proportional
  - Similar Symbol: ~



* same shape but not necessarily the same size

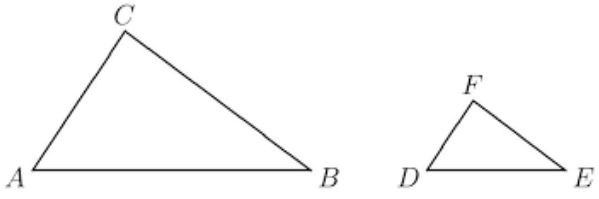
DCBA ~ SRQP

(order matters → match up corresponding angles)

EX: The two triangles are similar.

- List all pairs of congruent angles.

  $\angle A \cong \angle D$
  $\angle C \cong \angle F$
  $\angle B \cong \angle E$



- Write the ratios of the corresponding sides in a statement of proportionality.

  * Set up equal ratios of corresponding sides

$$\frac{AC}{DF} = \frac{CB}{FE} = \frac{BA}{ED}$$

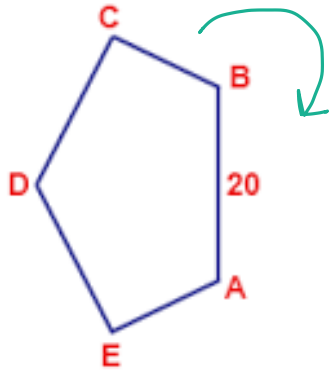
EX: EDCBA ~ TSRQP

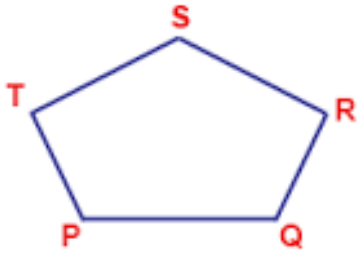
- List all pairs of congruent angles.

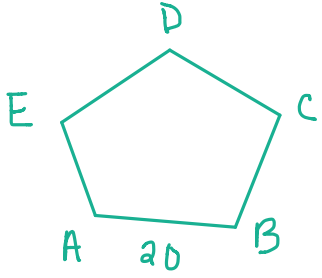
$\angle S \cong \angle D$
$\angle R \cong \angle C$
$\angle Q \cong \angle B$
$\angle P \cong \angle A$
$\angle T \cong \angle E$

- Write the ratios of the corresponding sides in a statement of proportionality.

$$\frac{SR}{DC} = \frac{RQ}{CB} = \frac{QP}{BA} = \frac{PT}{EA} = \frac{TS}{ED}$$

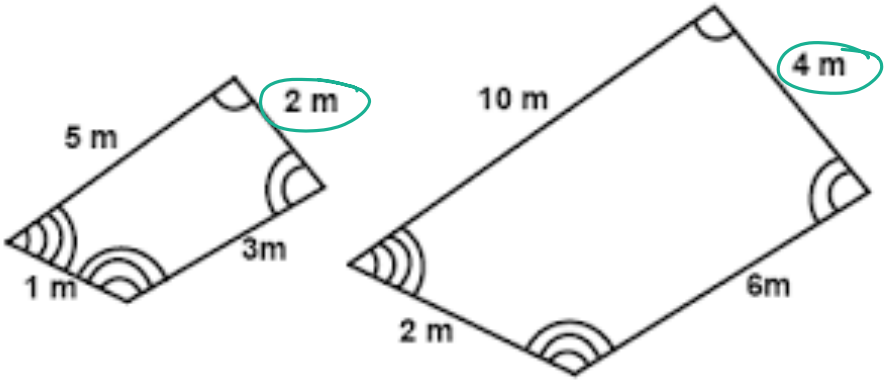






Scale Factor

- If two polygons are similar, the ratio of the lengths of two corresponding sides is called the scale factor.



Ratio of any 2 corresponding sides = scale factor

Ex: scale factor = $\frac{2}{4} = \boxed{\frac{1}{2}}$

(Notice – all sides would reduce to $\frac{1}{2}$)

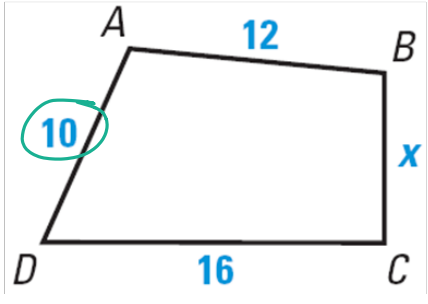
EX: Find the scale factor for each.

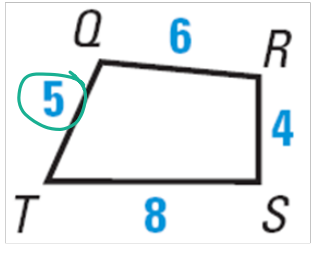
- ABCD to QRST

  * Pick any 2 corresponding sides.

  Scale Factor = $\frac{10}{5} = \boxed{\frac{2}{1}}$

In the diagram, $ABCD \sim QRST$.



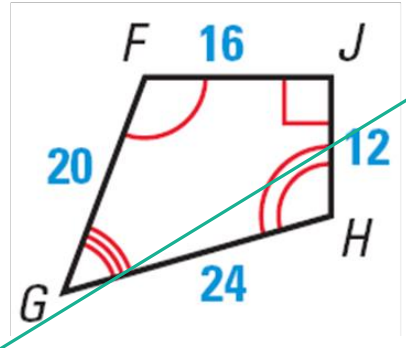


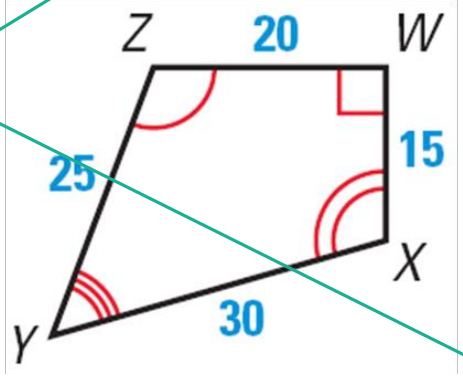
- QRST to ABCD

  Scale Factor = $\frac{5}{10} = \boxed{\frac{1}{2}}$

EX: Skip

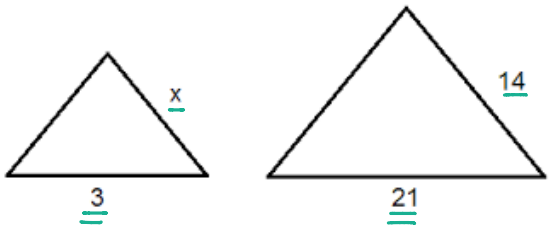
Determine whether the polygons are similar. If they are, write a similarity statement and find the scale factor of *ZYXW* to *FGHJ*.





Finding Missing Side Lengths in Similar Polygons

- Since similar polygons have sides that are proportional, you can use a proportion to solve for a missing side.
- To solve a proportion, use cross multiplication.



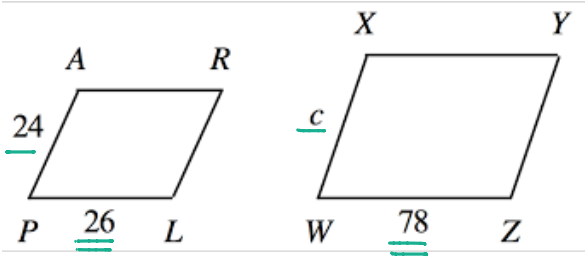
$\frac{x}{14} = \frac{3}{21}$

$\frac{x}{14} \times \frac{3}{21}$

$\frac{21x}{21} = \frac{42}{21}$

$x = 2$

EX: Solve for c.



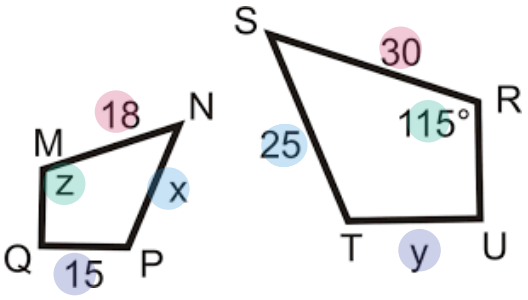
$\frac{24}{c} = \frac{26}{78}$

$\frac{24}{c} \times \frac{26}{78}$

$\frac{26c}{26} = \frac{1872}{26}$

$c = 72$

EX: Solve for x, y, and z.



Angle: $z = 115°$

Sides: $\frac{x}{25} = \frac{18}{30}$

$\frac{30x}{30} = \frac{450}{30}$

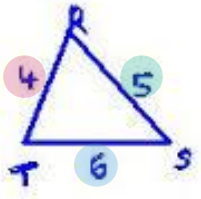
$x = 15$

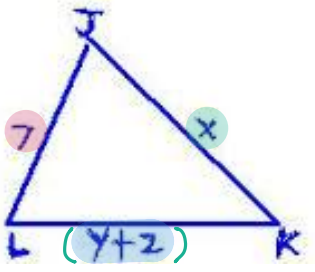
$\frac{y}{15} = \frac{30}{18}$

$\frac{18y}{18} = \frac{450}{18}$

$y = 25$

EX: Solve for x and y.





Put any sums or differences in parantheses so you remember to distribute!

$$\frac{x}{5} = \frac{7}{4}$$

$$\frac{4x}{4} = \frac{35}{4}$$

$$\boxed{x = 8.75}$$

$$\frac{(y+2)}{6} = \frac{7}{4}$$

$$4(y+2) = 42$$

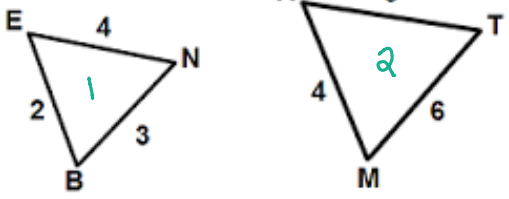
$$4y + 8 = 42$$
$$-8 \quad -8$$

$$\frac{4y}{4} = \frac{34}{4}$$

$$\boxed{y = 8.5}$$

Perimeters

- All sides added up.
- If two polygons are Similar, the ratio of their perimeters is equal to the ratio of sides
- Both are also equal to the scale factor of the polygons.



$P_1 = 4 + 3 + 2 = 9$

$P_2 = 8 + 6 + 4 = 18$

Perimeter Ratio $= \frac{9}{18} = \frac{1}{2}$

Side Ratio $= \frac{4}{8} = \frac{1}{2}$

Scale Factor $= \frac{1}{2}$

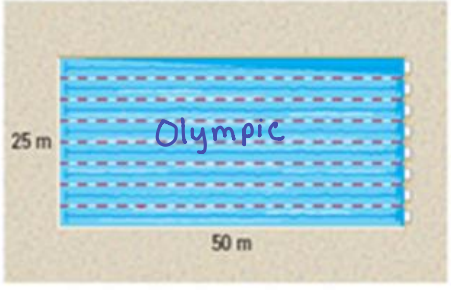
All equal

EX:

EXAMPLE 4 **Find perimeters of similar figures**

Swimming

A town is building a new swimming pool. An Olympic pool is rectangular with length 50 meters and width 25 meters. The new pool will be similar in shape, but only 40 meters long.



x New
40

a. Find the scale factor of the new pool to an Olympic pool.

$SF = \frac{\text{New}}{\text{Olym.}} = \frac{40}{50} = \frac{4}{5}$

EXAMPLE 4 Find perimeters of similar figures

b. Find the perimeter of an Olympic pool and the new pool.

* Ratio of Perimeters = Scale Factor

* Olympic Perimeter = 25 + 50 + 25 + 50 = 150
  New Pool Perimeter = x

* $\frac{\text{New}}{\text{Olympic}}$: $\frac{x}{150} = \frac{4}{5}$

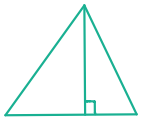
$$\frac{5x}{5} = \frac{600}{5}$$

$$\boxed{x = 120 \text{ m}}$$

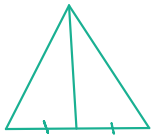
Corresponding Lengths in Similar Polygons

- If two polygons are Similar, then the ratio of any two corresponding lengths in the polygons is equal to the scale factor of the polygons.
  - Examples:

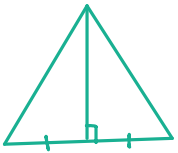
Altitudes:



Median:



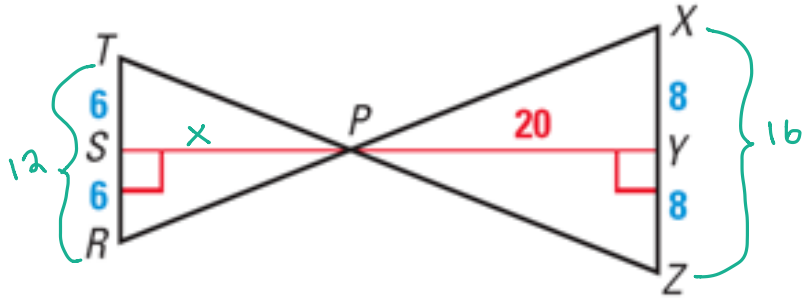
Perpendicular Bisector:



EX:

EXAMPLE 5 **Use a scale factor**

In the diagram, $\Delta TPR \sim \Delta XPZ$. Find the length of the altitude $\overline{PS}$.



$$\frac{x}{20} = \frac{12}{16}$$

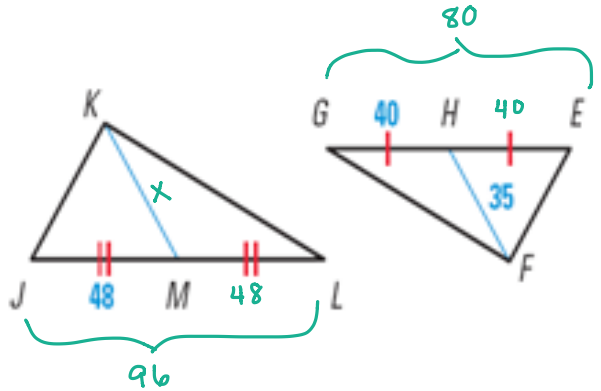
$$\frac{16x}{16} = \frac{240}{16}$$

$$\boxed{x = 15}$$

EX:

GUIDED PRACTICE for Example 5

7. In the diagram, $\Delta JKL \sim \Delta EFG$. Find the length of the median $\overline{KM}$.



$\frac{x}{35} = \frac{96}{80}$

$\frac{80x}{80} = \frac{3360}{80}$

$x = 42$