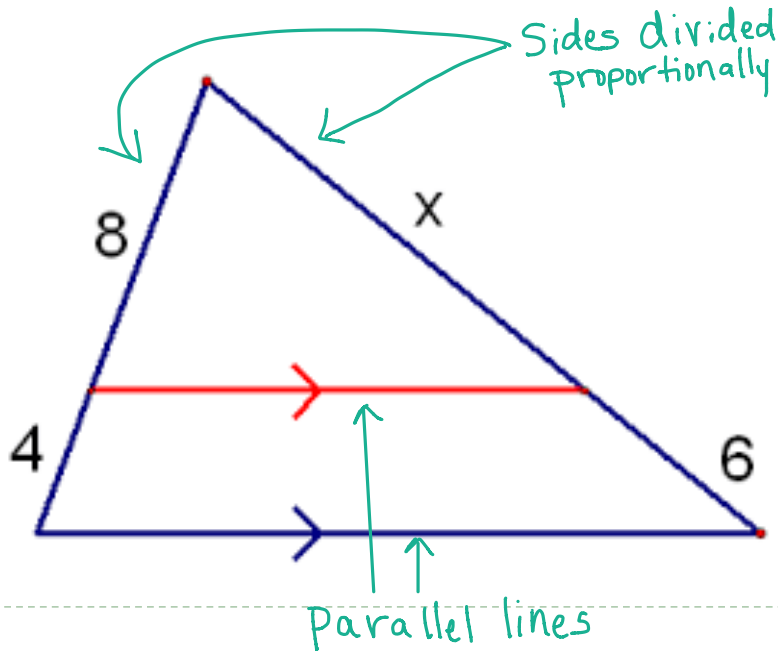


6.5

Use Proportionality Theorems

Triangle Proportionality Theorem

- If a line is parallel to one side of a triangle intersects the other two sides, then it divides the two side proportionally.

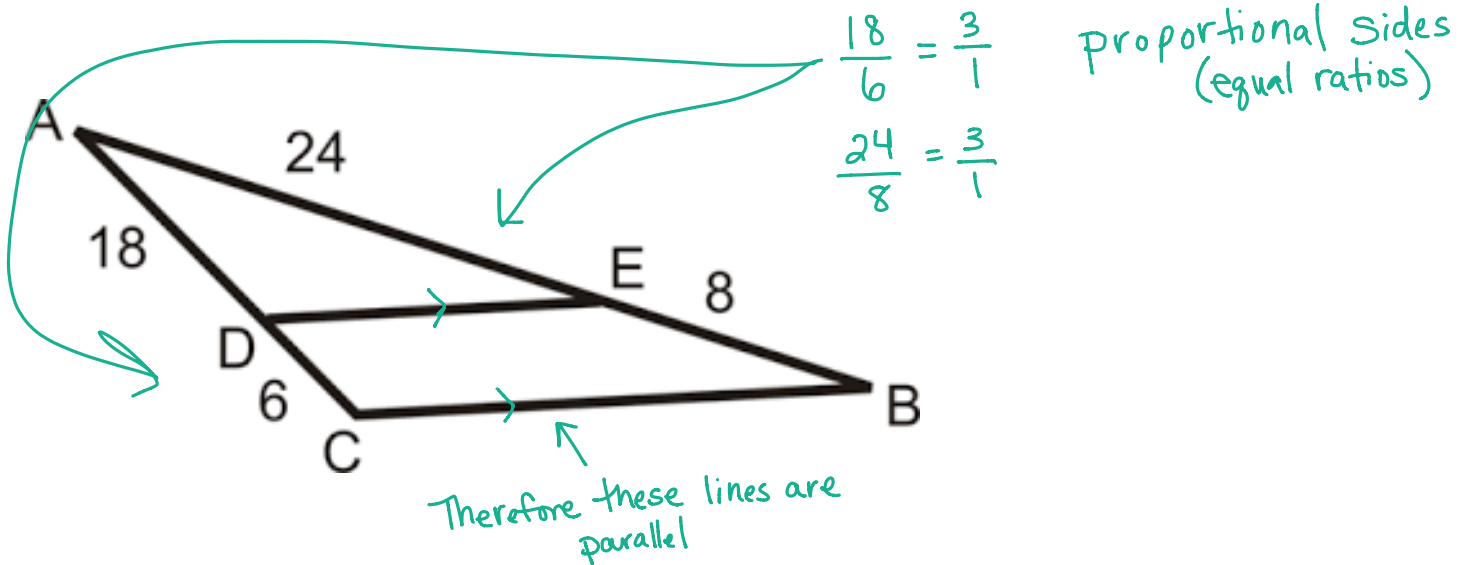


Set up a proportion:

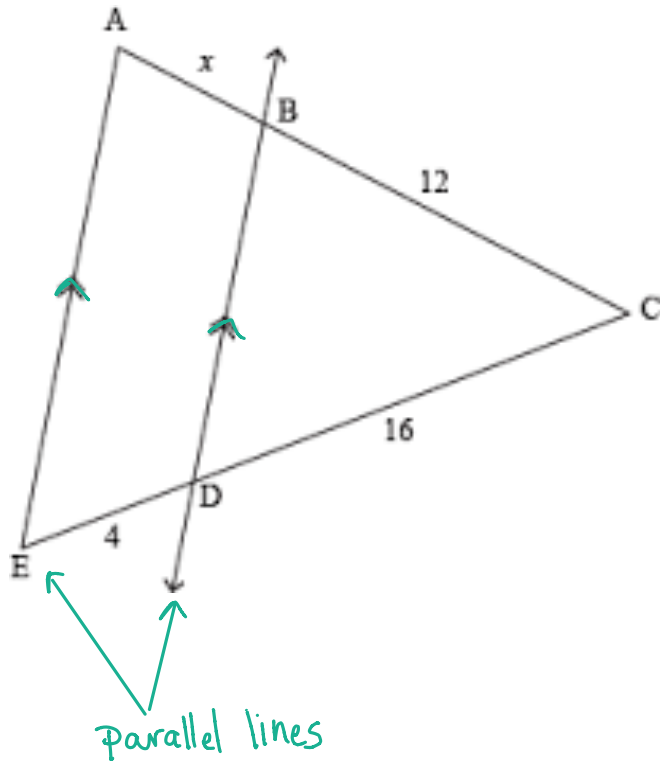
$$\frac{8}{4} = \frac{x}{6}$$

Converse of the Triangle Proportionality Theorem

- If a line divides 2 sides of a triangle proportionally, then it is parallel to the 3rd side.



EX: Find x.



Proportion:

$$\frac{x}{12} = \frac{4}{16}$$

$$\frac{x}{12} \times \frac{4}{16}$$

$$\frac{16x}{16} = \frac{48}{16}$$

$$\boxed{x = 3}$$

EX:

In the diagram, $\overline{QS} \parallel \overline{UT}$, $RS = 4$, $ST = 6$, and $QU = 9$. What is the length of \overline{RQ} ?

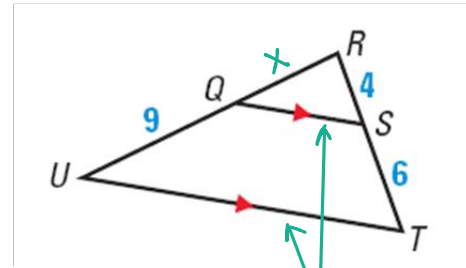
Proportion:

$$\frac{x}{9} = \frac{4}{6}$$

$$\frac{x}{9} \times \frac{4}{6}$$

$$\frac{6x}{6} = \frac{36}{6}$$

$$\boxed{x = 6}$$



parallel lines



EX:

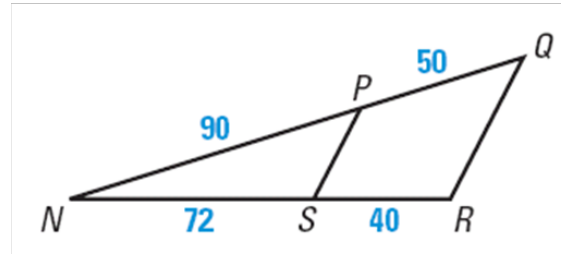
2. Determine whether $\overline{PS} \parallel \overline{QR}$.

Set up ratios:

$$\frac{90}{50} = \frac{9}{5}$$

$$\frac{72}{40} = \frac{9}{5}$$

Equal ratios so $\overline{PS} \parallel \overline{QR}$



EX:

Shoerack

On the shoerack shown,
 $AB = 33$ cm, $BC = 27$ cm,
 $CD = 44$ cm, and $DE = 25$ cm,
Explain why the gray shelf is not parallel to the floor.

Set up ratios :

$$\frac{27}{33} = \frac{9}{11} \approx 0.82$$

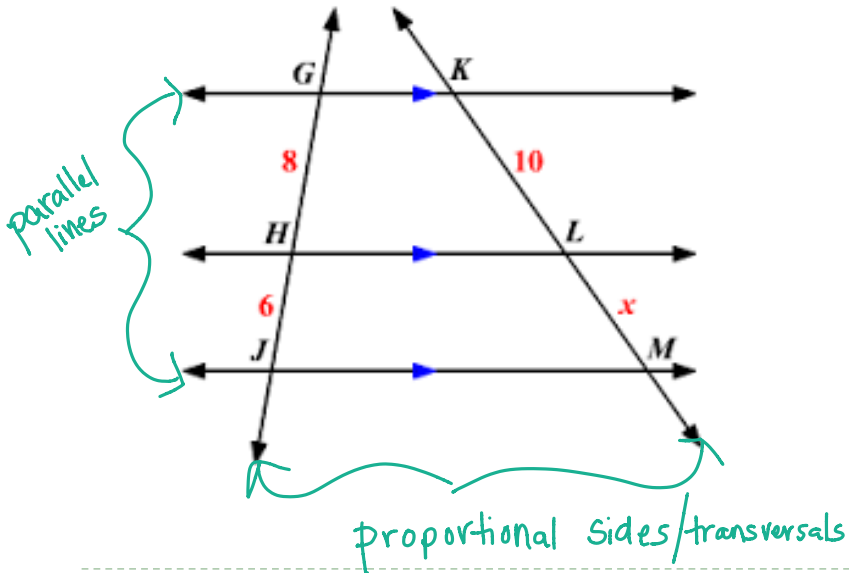
$$\frac{44}{25} \approx 1.76$$

Ratios are not equal - sides are not proportional - shelf is not parallel to the floor



Parallel Lines Theorem

- If 3 parallel lines intersect 2 transversals, then they divide the transversals proportionally.

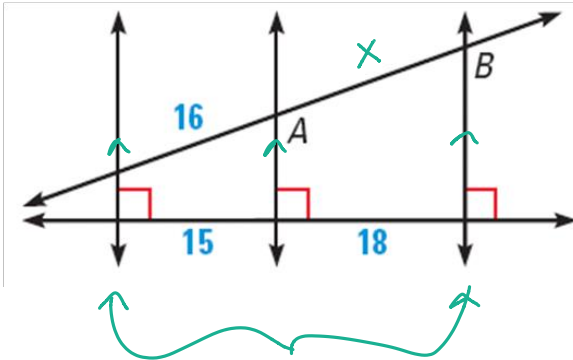


Proportion

$$\frac{8}{6} = \frac{10}{x}$$

EX: Find the length of AB.

3.



Parallel lines because
corresponding angles
are equal

Proportion:

$$\frac{X}{16} = \frac{18}{15}$$

$$\frac{X}{16} \times \frac{15}{15}$$

$$\frac{15X}{15} = \frac{288}{15}$$

$$X = 19.2$$

EX:

City Travel

In the diagram, $\angle 1$, $\angle 2$, and $\angle 3$ are all congruent and $GF = 120$ yards, $DE = 150$ yards, and $CD = 300$ yards. Find the distance HF between Main Street and South Main Street.

Solve for x first:

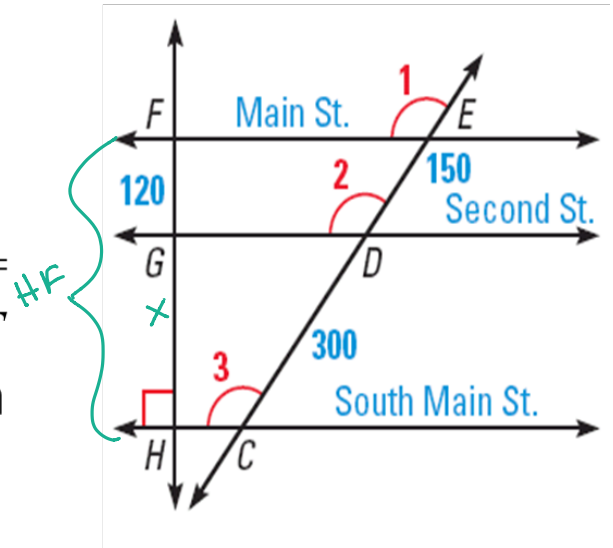
$$\frac{x}{120} = \frac{300}{150}$$

$$\frac{x}{120} \times \frac{150}{150} = \frac{300}{150}$$

$$\frac{150x}{150} = \frac{36,000}{150}$$
$$x = 240$$

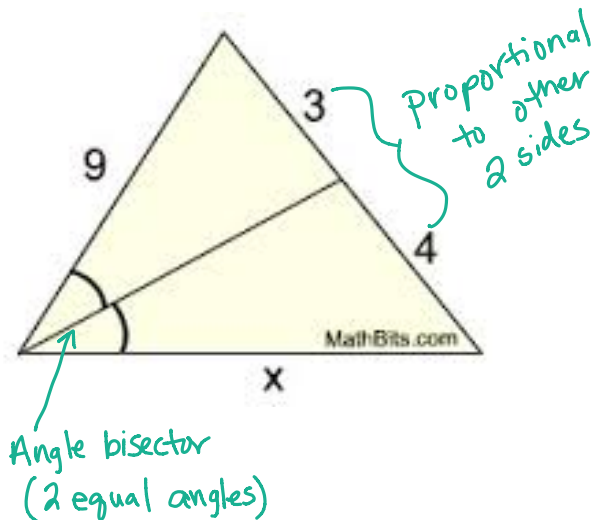
$$HF = 240 + 120$$

$$HF = 360$$



Angle Bisector Theorem

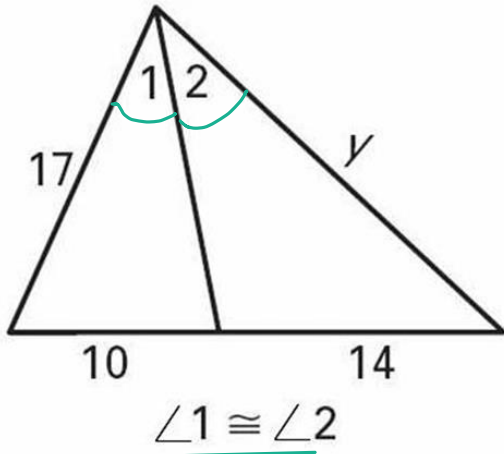
- If a ray bisects an angle of a triangle, then it divides the opposite side into segments whose lengths are proportional to the lengths of the two other sides.



Proportion:

$$\frac{3}{9} = \frac{4}{x}$$

EX: Find the value of the variable.



Proportion:

$$\frac{10}{17} = \frac{14}{y}$$

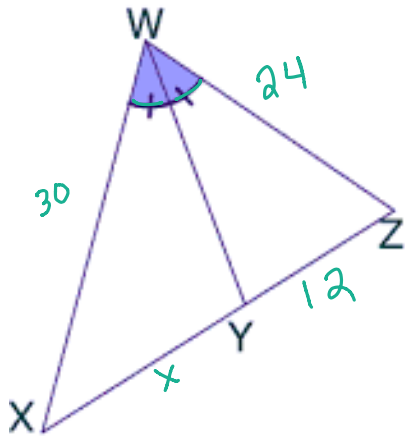
$$\frac{10}{17} \neq \frac{14}{y}$$

$$\frac{10y}{10} = \frac{238}{10}$$

$$y = 23.8$$



EX:



$$\overline{WZ} = 24$$

$$\overline{ZY} = 12$$

$$\overline{WX} = 30$$

$$\overline{XY} = ?$$

Proportion:

$$\frac{x}{30} = \frac{12}{24}$$

$$\frac{x}{30} \neq \frac{12}{24}$$

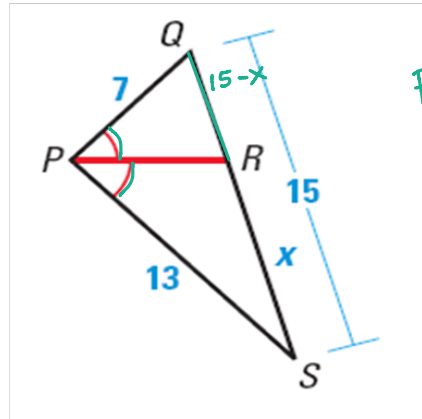
$$\frac{24x}{24} = \frac{360}{24}$$

$$\boxed{x = 15}$$



EX:

In the diagram, $\angle QPR \cong \angle RPS$. Use the given side lengths to find the length of \overline{RS} .



Proportion:

$$\frac{x}{13} = \frac{15-x}{7}$$

$$\frac{x}{13} \neq \frac{(15-x)}{7}$$

$$7x = 13(15-x)$$

$$7x = 195 - 13x$$
$$+13x \quad +13x$$

$$\frac{20x}{20} = \frac{195}{20}$$

$$x = 9.75$$