# 6.2
# Confidence Intervals for the Mean (with unknown σ)

* used when pop. standard deviation is not known

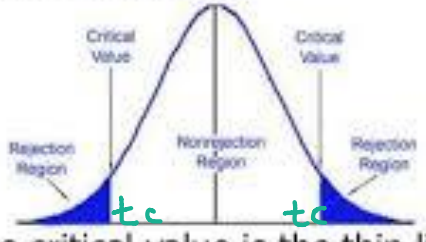
# The t-Distribution

* Used to construct a confidence interval (a range of values) for a population mean when the population standard deviation (σ) is not known.
* Critical values are $t_c$:

* Find $t_c$ using your calculator:
  - Distr
  - InvT
    area:
    df:

**Definition**

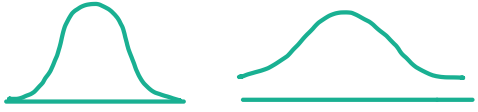
A critical value is the number on the borderline separating sample statistics that are likely to occur from those that are unlikely to occur.




The critical value is the thin line between rejection and acceptance.

# Properties of the t-Distribution

* 1) The Mean, median, and mode are equal to Zero.
* 2) Bell-shaped and symmetric about the mean.
* 3) Area under the curve equals one.
* 4) Tails are "thicker" than in the standard normal distribution.



Normal t-distribution

# Cont.

* 5) Standard deviation is greater than one (more spread out).
* 6) Family of curves determined by the degree of freedom (df).
  * The number of free choices left after a sample statistic is calculated.
  * D.f. = n - 1

# Cont.

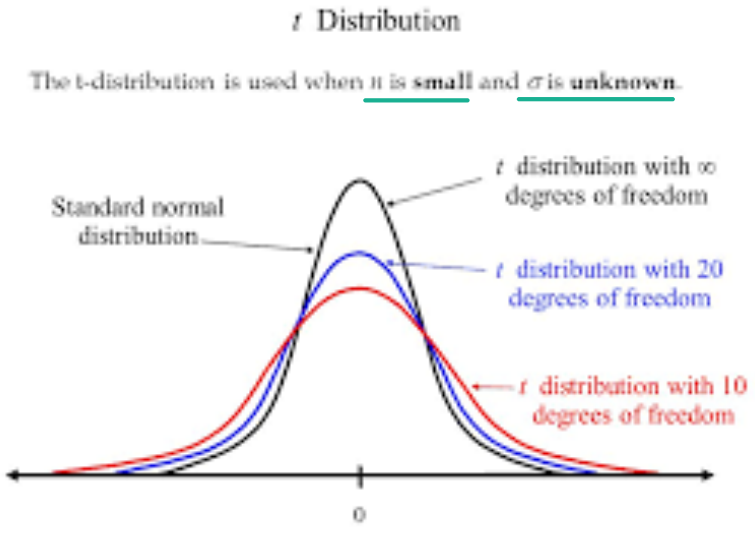
* Degrees of freedom illustration:
  * 25 Students in a class
  * 25 Chairs in the classroom
  * Each of the first 24 Students to enter the classroom has a choice as to which chair they will sit in. There is no freedom or choice, however, for the 25th student who enters the room.

* 24 degrees of freedom
  - 24 choices

* df = n-1 = 25-1 = 24

# Cont.

* 7) As the degrees of freedom increases, the t-distribution approaches the Standard Normal distribution.



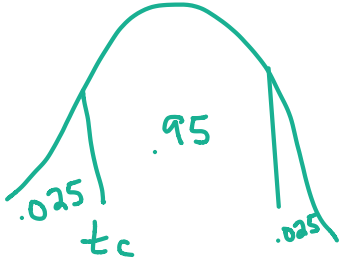
# EX:

* Find the critical value $t_c$ for a 95% confidence level when the sample size is 15.

$C = 0.95$ ⟶

$n = 15$

$df = 15 - 1 = 14$



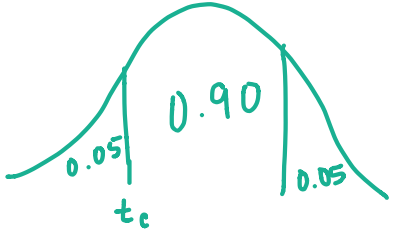

xDistr
* InvT
area : 0.025
df : 14

$t_c = 2.145$

# EX:

* Find the critical value $t_c$ for a 90% confidence level when the sample size is 22.

$c = 0.90$
$n = 22$
$df = 22 - 1 = 21$




* Distr
* InvT
  area: 0.05
  df : 21

$t_c = 1.721$

# Constructing a Confidence Interval for a Population Mean (σ unknown)

* 1) Find the sample stats:

  $n$ = sample size
  $\bar{x}$ = sample mean
  $s$ = sample standard deviation

* 2) Identify the degrees of freedom $(n-1)$,
  the level of confidence $(C)$,
  and the critical value $(t_c)$:

  $C$ = %
  $df = n-1$
  $t_c$ = use calculator (like in previous 2 examples)

# Cont.

* 3) Find the Margin of error (E):

$$E = t_c\left(\frac{s}{\sqrt{n}}\right)$$

* 4) Find interval by adding and subtracting E to the sample mean ($\bar{x}$):

$$\bar{x} - E < \mu < \bar{x} + E$$

$\mu$: population mean