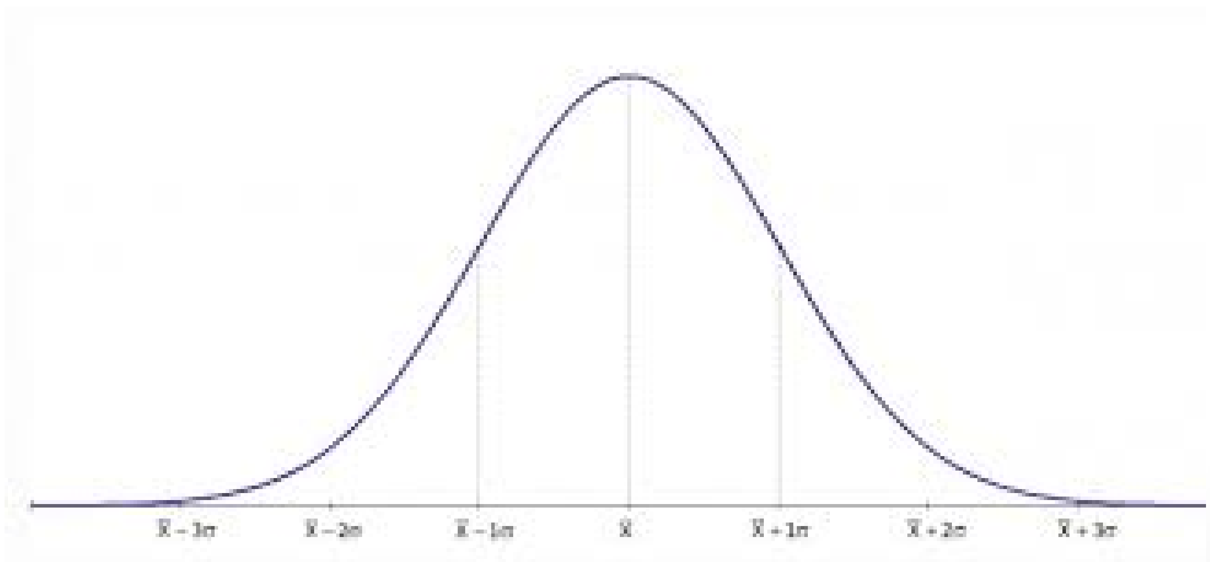
Chapter 5
Normal Probability Distributions

5.1
INTRODUCTION TO NORMAL DISTRIBUTIONS AND THE STANDARD NORMAL DISTRIBUTION

Normal Distribution

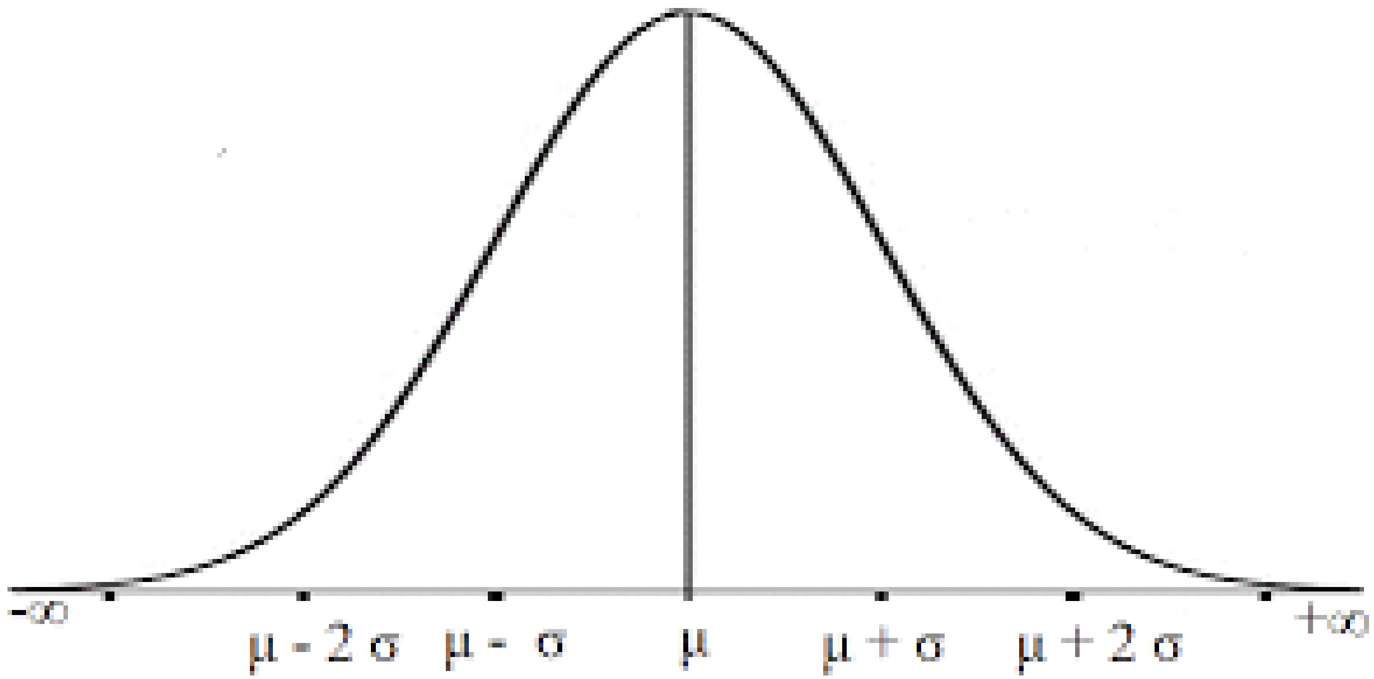
- A ___________________________ probability distribution for a ____.
- **Continuous** – has an __________________________ of possible values
- **Graph** is called the _______________________________.



Normal Distribution Properties

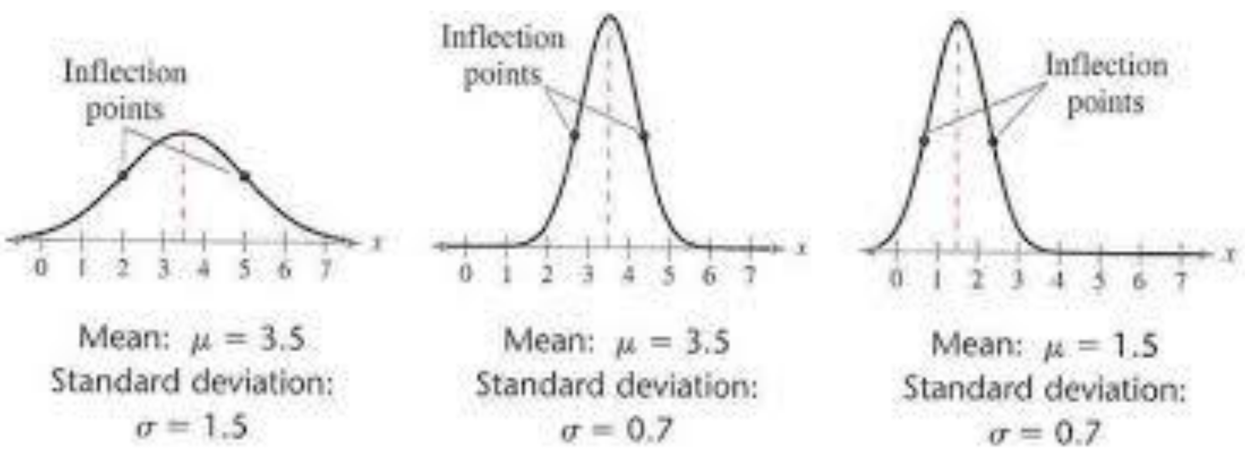
- 1) The ____ are all _____________________ (in the middle).
- 2) ___________________ and ____________________
- 3) Area under the curve equals ________________
- 4) Curve approaches, but ________________ touches, the x-axis.
- 5) Has two ______________________________: the points at which the curve changes from curving ________________ to curving ____________________
    - These points are ___________________________ away from the ____________________.

Properties Cont.



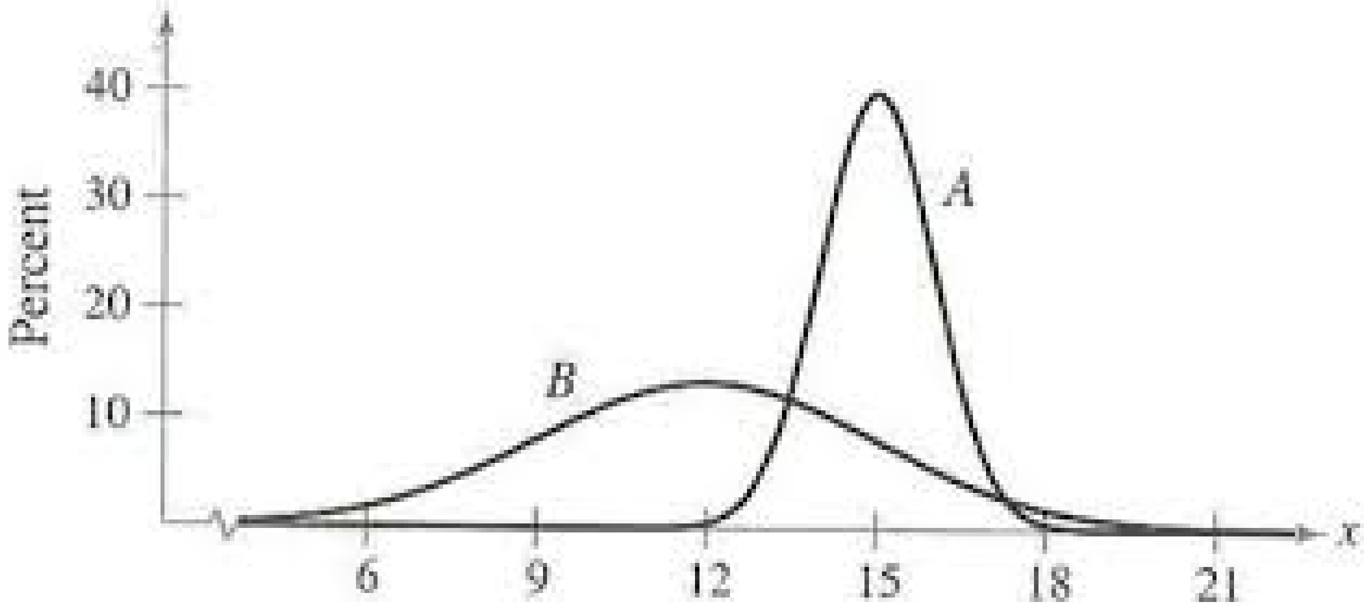
Mean and Standard Deviation

- The mean and standard deviation determine the ______________________ of the normal curve.
- The mean gives the location of the ___________________ _____________________________________.
- The standard deviation describes how the data is _______________________________.
  - Larger SD = ____________________________________
  - Smaller SD = ____________________________________



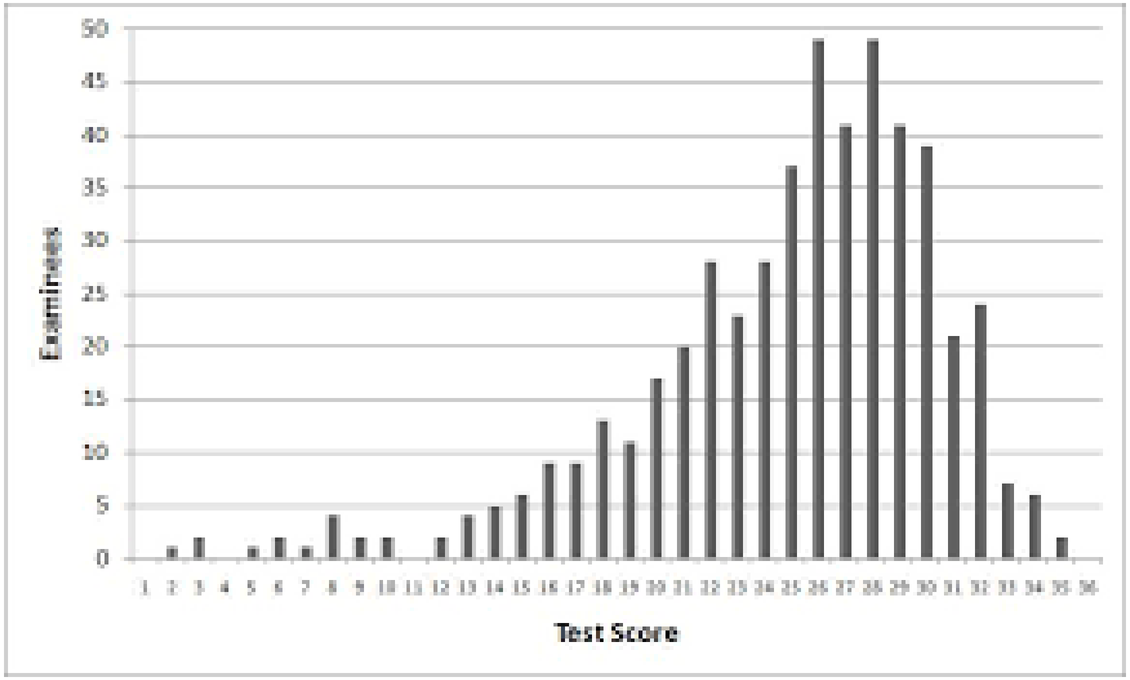
EX:

- Which normal curve has a greater mean?
- Which normal curve has a greater standard deviation?

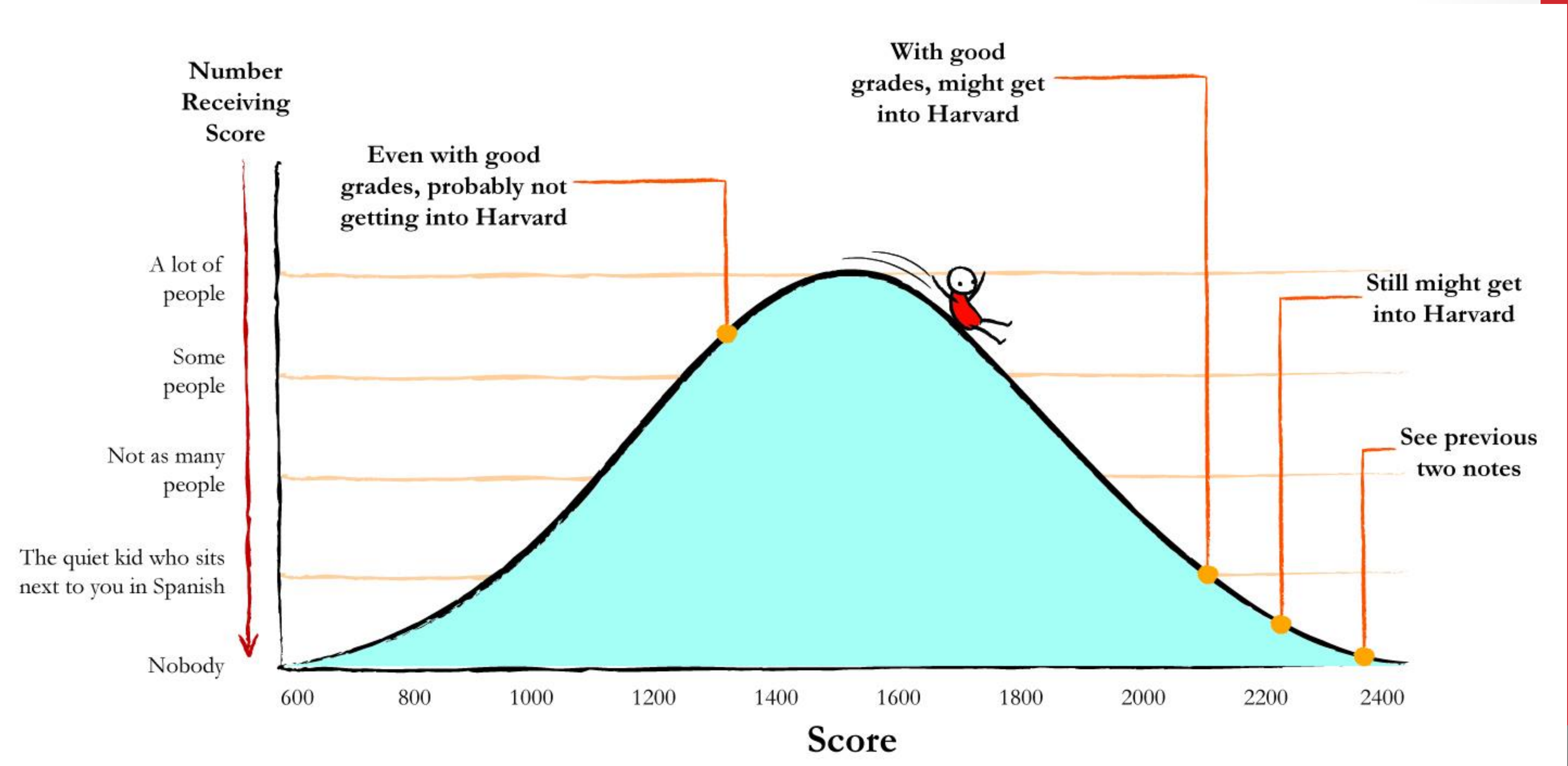


EX:

- Does the graph represent a normal distribution? Explain.
- If it does, estimate the mean and standard deviation.



EX:

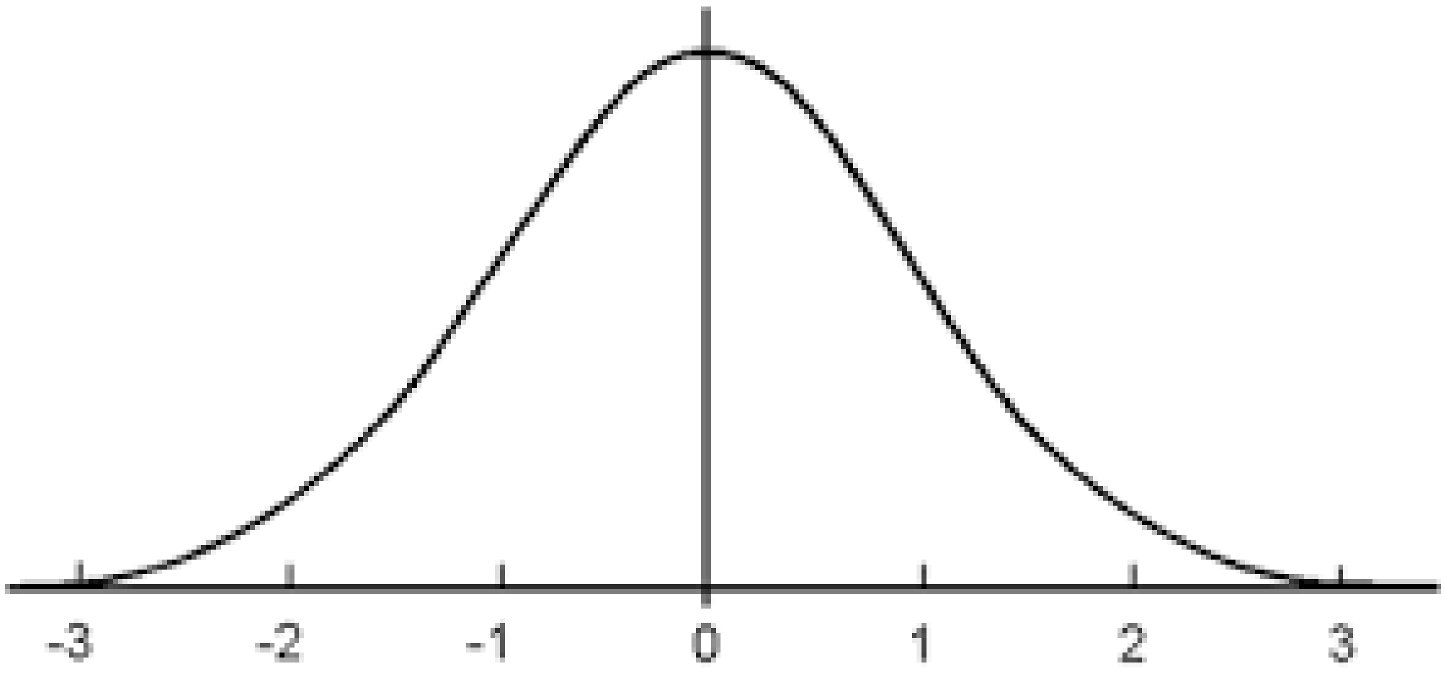


Number Receiving Score
Even with good grades, probably not getting into Harvard
With good grades, might get into Harvard
Still might get into Harvard
See previous two notes
A lot of people
Some people
Not as many people
The quiet kid who sits next to you in Spanish
Nobody
600
800
1000
1200
1400
1600
1800
2000
2200
2400
Score

The Standard Normal Distribution

- A normal distribution with a ______________________________ and a ____.
- Horizontal scale is corresponds to ________________________.
  - Z-score - The number of _________________________________ a value lies from the ______________________________.
  - EX: z-score = 1 ____________________________________
- All normal distributions can be converted to standard normal distributions by converting _________ into a _______________
  - Formula to turn an x-value into a z-score:
- Area under the curve = ___________________________

The Standard Normal Distribution

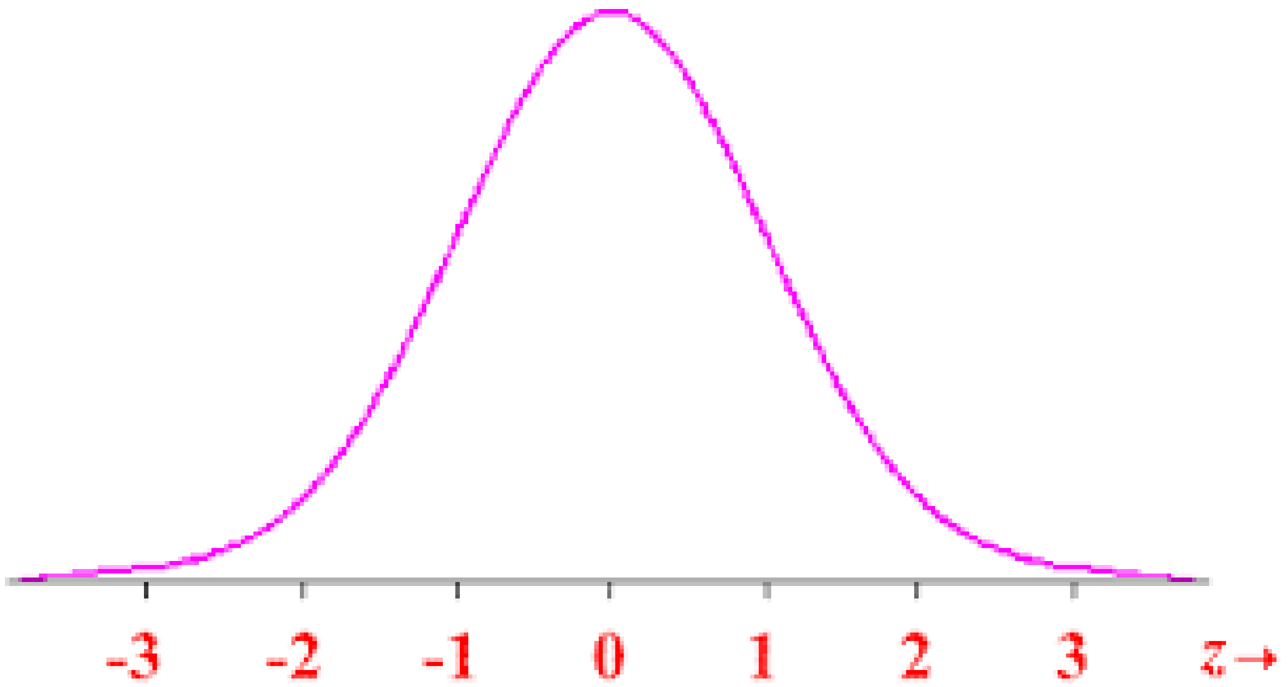


Finding the Area Under the Standard Normal Distribution

- Use the formula to ____________________ an ________________ into a ____________________________.
  - Round to the nearest ____________________________.
- Use your calculator:
  - **DISTR**
  - **normalcdf**
  - **Lower limit:**
  - **Upper limit:**
  - **Mean =**
  - **Standard deviation =**

- **Notice:**
- Areas for z-scores farther left are __________________________
- Area for z = 0 is _________________________________
- Areas for z-scores farther right are _________________________

Areas Cont.



EX:

- Find the cumulative area that corresponds to a z-score of 1.15.

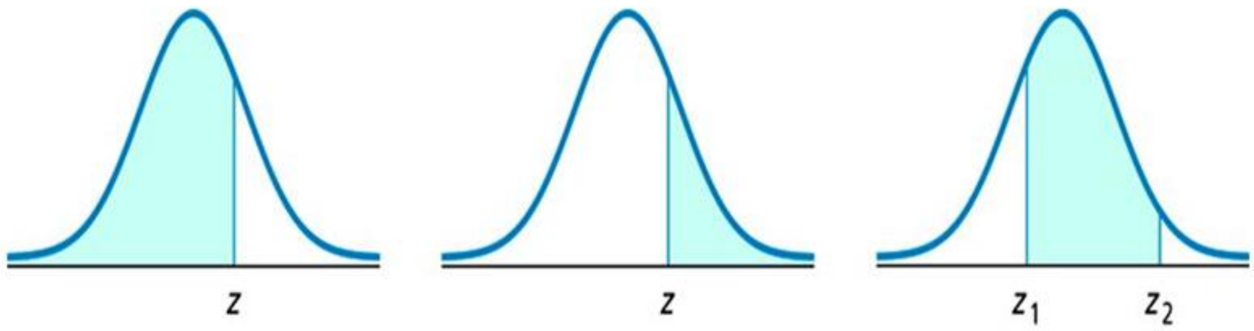
- Find the cumulative area that corresponds to a z-score of -0.24.

- Find the cumulative area that corresponds to a z-score of 2.19.

Guidelines for Finding Area Under the Standard Normal Curve

- 1) _________________ the curve and ___________________ the appropriate area, if necessary.
- 2) Find the area of the corresponding _____________________ and _____________________, if necessary.
- See next slide.

Using Table A to find the area under the standard normal curve that lies (a) to the left of a specified z-score, (b) to the right of a specified z-score, and (c) between two specified z-scores

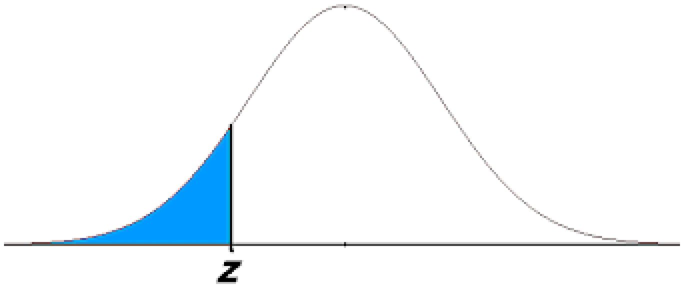


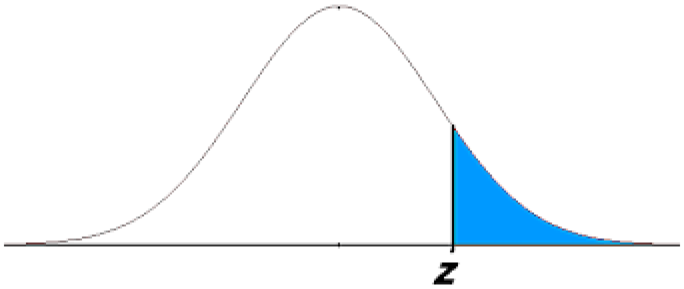
(a) Shaded area: Area to left of z

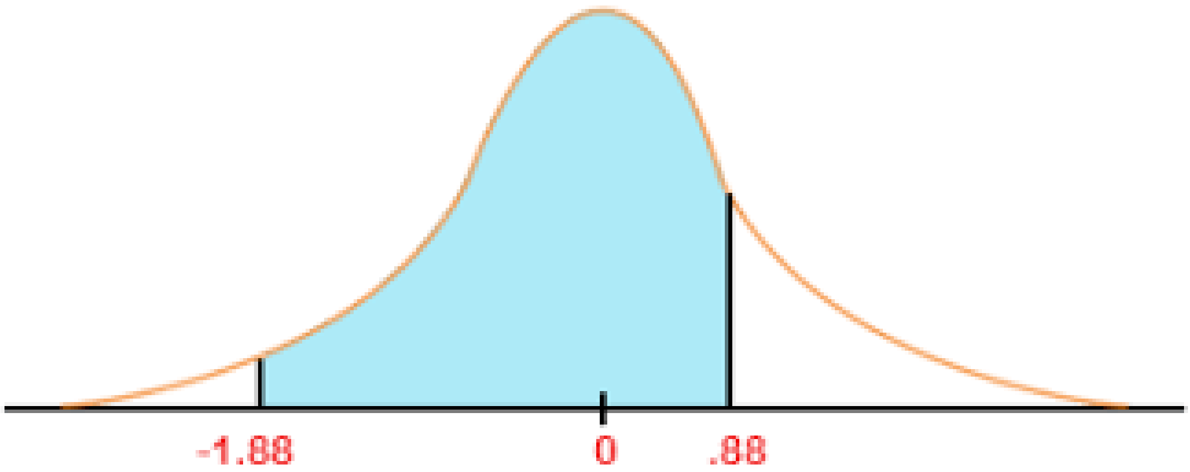
(b) Shaded area: $1 - (\text{Area to left of } z)$

(c) Shaded area: $(\text{Area to left of } z_2) - (\text{Area to left of } z_1)$

EX: Find the area in the indicated region.






-1.88
0
.88

EX: Find the area.

- To the left of $z = -1.15$ and to the right of 1.87.

5.2 Normal Distributions: Finding Probabilities

For a Normal Distribution:

- The ___________________________ that _______ will lie in an interval can be found by finding the ________________ under the normal curve.
- However, before finding the area – you must ______________ all _________________ to their corresponding __________________________ using:
- Then use your ________________________ to find the _________________ under the _________________________ ____ ____.

EX:

- A survey indicates that people use their cell phone an average of 1.5 years before buying a new one. The standard deviation is 0.25 year. A cell phone user is selected at random. Find the probability that the user will keep his or her current phone for less than 1 year before buying a new one.

EX:

- A survey indicates that for each trip to a supermarket, a shopper spends an average of 45 minutes with a standard deviation of 12 minutes in the store. A shopper enters the store.
- A) Find the probability that the shopper will be in the store for each interval listed below.
- B) Interpret your answer when 200 shoppers enter the store. How many shoppers would you expect to be in the store for each interval of time?

Between 24 and 54 minutes.

- More than 39 minutes

5.3 Normal Distributions: Finding Values

To find a **value** - given an **area** or **probability**:

- Find the ________________________ using your __________________________:
  - DISTR
  - invNorm
  - Area: Enter the given ____________________ or _____________________ (remember they are ___________)
  - Mean:
  - Standard Deviation:
- If asked, ______________________ the ___________________ into an _______________________ using:

EX:

- Find a z-score that corresponds to a cumulative area of 0.3632.

- Find the z-score that has 10.75% of the distribution's area to its right.

- Find the z-score for which 95% of the distribution's area lies between –z and z.

EX:

- Find the z-score that corresponds to each percentile.
- P_5
- P_{50}

EX:

- A vet records the weights of cats treated at a clinic. The weights are normally distributed, with a mean of 9 pounds and a standard deviation of 2 pounds. Find the weights x corresponding to z-scores of 1.96, -0.44, and 0.

EX:

- Scores for a California Standardized Test are normally distributed, with a mean of 50 and a standard deviation of 10. A college will only accept applicants with scores in the top 10%. What is the lowest score an applicant can earn and still be eligible to be accepted by the college?

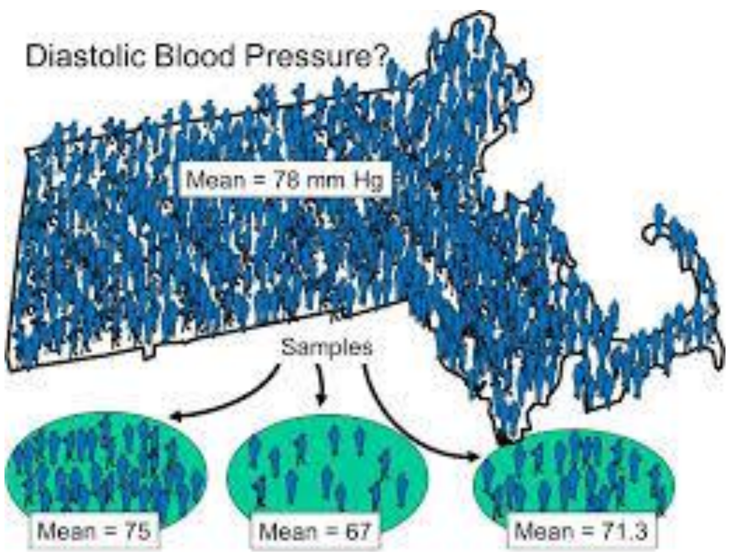
EX:

- In a randomly selected sample of women ages 20-34, the mean total cholesterol level is 181 milligrams per deciliter with a standard deviation of 37.6 milligrams per deciliter. Find the highest total cholesterol level a woman in this 20-34 age group can have and still be in the bottom 1%.

5.4
Sampling Distributions and the Central Limit Theorem

Sampling Distribution

- When ________________________ of _________________ are repeatedly taken from a ______________________________.
- REMEMBER: WE use characteristics of a __________________ __________________________ to estimate characteristics of a ____.
- EX: You find the __________________ of several samples to estimate the ______________________________________.



Sampling Distributions of Sample Means

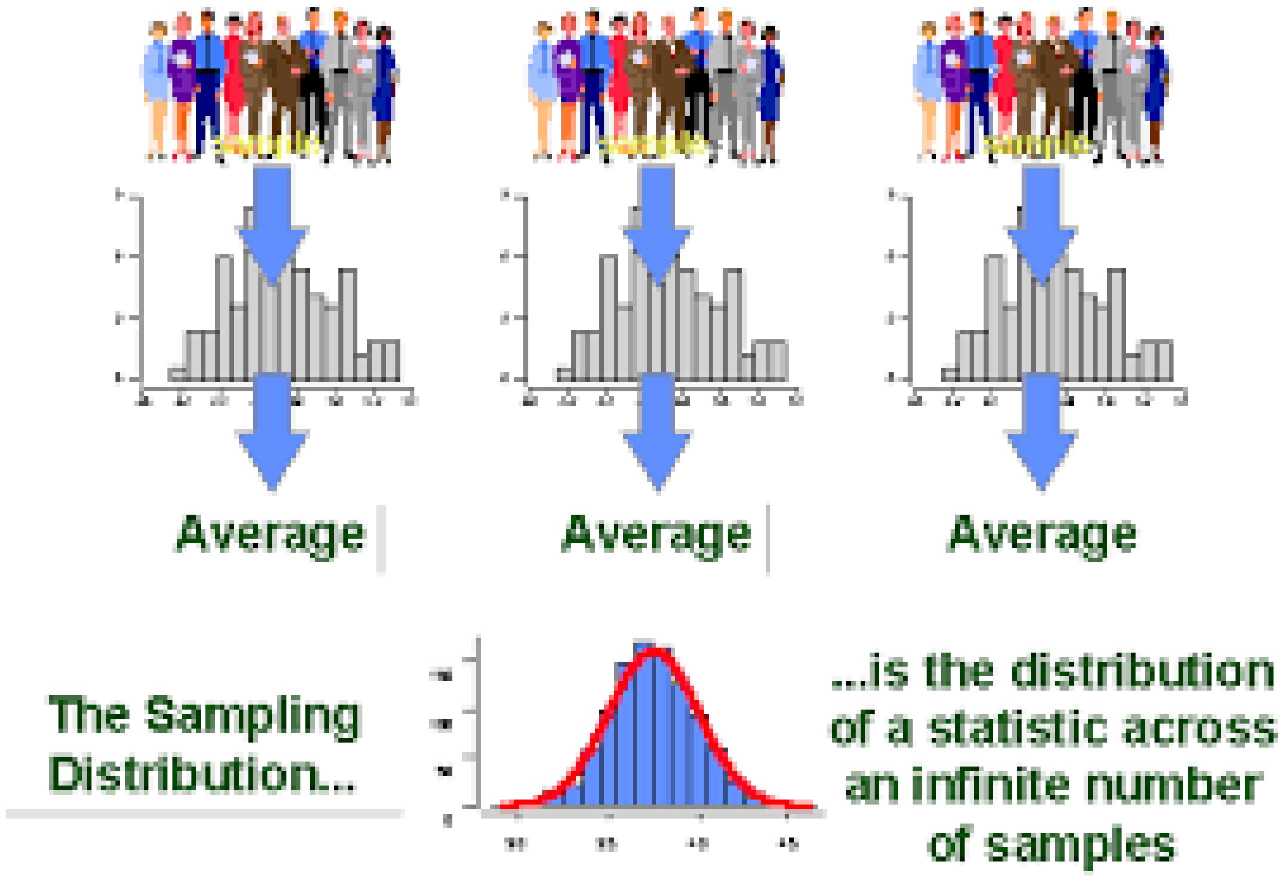
- When the _________________________ of several ______________________ is taken.
- **Properties:**
- When the ______________________ of all of the ________________________________ is calculated, it _____________________ the ______________________________:

Cont.

- The ______________________________of the __________________ ______________________ is ____________________ to the ____________________________________ of the ____________________________________ divided by ____:

- The standard deviation of the __________________________ is called the ____.

Sampling Distributions



EX:

- List all possible samples of size n = 2, with replacement, from the population {1, 3, 5, 7}. Calculate the mean of each sample. Find the mean, variance, and standard deviation of the sample means. Compare your results with the population results: mean = 4, variance = 5, standard deviation = √5 = 2.236

Sample	Mean

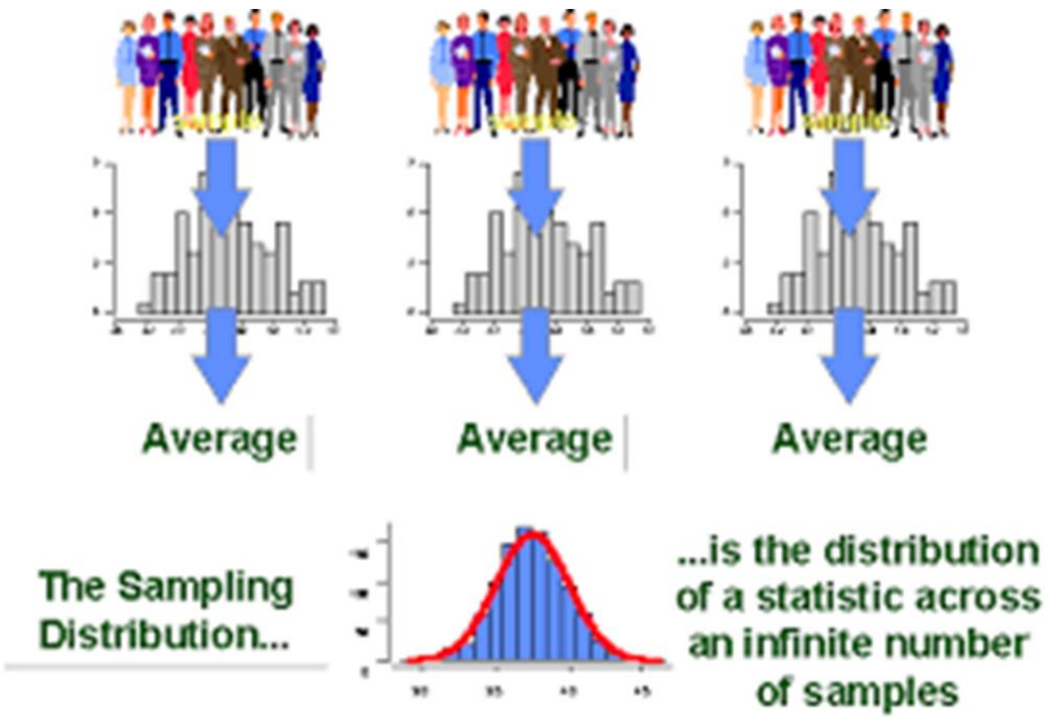
Sample	Mean

The Central Limit Theorem

- Describes the relationship between the ____________________ ________________________________ and the __________________________________ that the sample is taken from.
- Provides the information you will need to use _____________ _____________________to make ___________________________ about a ____.

The Central Limit Theorem

- 1) If sample sizes of ________________________________, are drawn from **any population** with ____________________ and ________________________________, then the sampling distribution is a ____________________________ (even if the population is not).



Cont.

- 2) If the population itself is ____________________________, then the sampling distribution is ________________________ ___________________________ for _______________ sample size __________.

For EITHER situation:

Sample Mean = population mean:

Sample Variance =

Sample Standard Deviation =

EX:

- Cell phone bills for residents of a city have a mean of $47 and a standard deviation of $9. Random samples of 100 cell phone bills are drawn from this population, and the mean of each sample is determined. Find the mean and standard deviation of the sampling distribution of sample means. Then sketch a graph of the sampling distribution.

Probability and the Central Limit Theorem

- To find the ______________________________ that a ________________________________ will lie in a given ___________________________:
- Change the given ______________ to a ____________________ using:

- Find the corresponding ____________________ using your calculator and ____________________________ is necessary.

EX:

- The average time spent driving each day for an age group of 15-19 is 25 minutes. You randomly select 50 drivers ages 15-19. What is the probability that the mean time they spend driving each day is between 24.7 and 25.5 minutes? Assume that the standard deviation is 1.5 minutes.

EX:

- The mean room and board expense per year at four-year colleges is $9126. You randomly select 9 four-year colleges. What is the probability that the mean room and board is less than $9400? Assume the standard deviation is $1500.

EX:

- The average credit card debt carried by undergraduates is normally distributed with a mean of \$3173 and a standard deviation of \$1120. You randomly select 25 undergraduates who are credit card holders. What is the probability that their mean credit card balance is less than \$2700. Is this considered unusual?

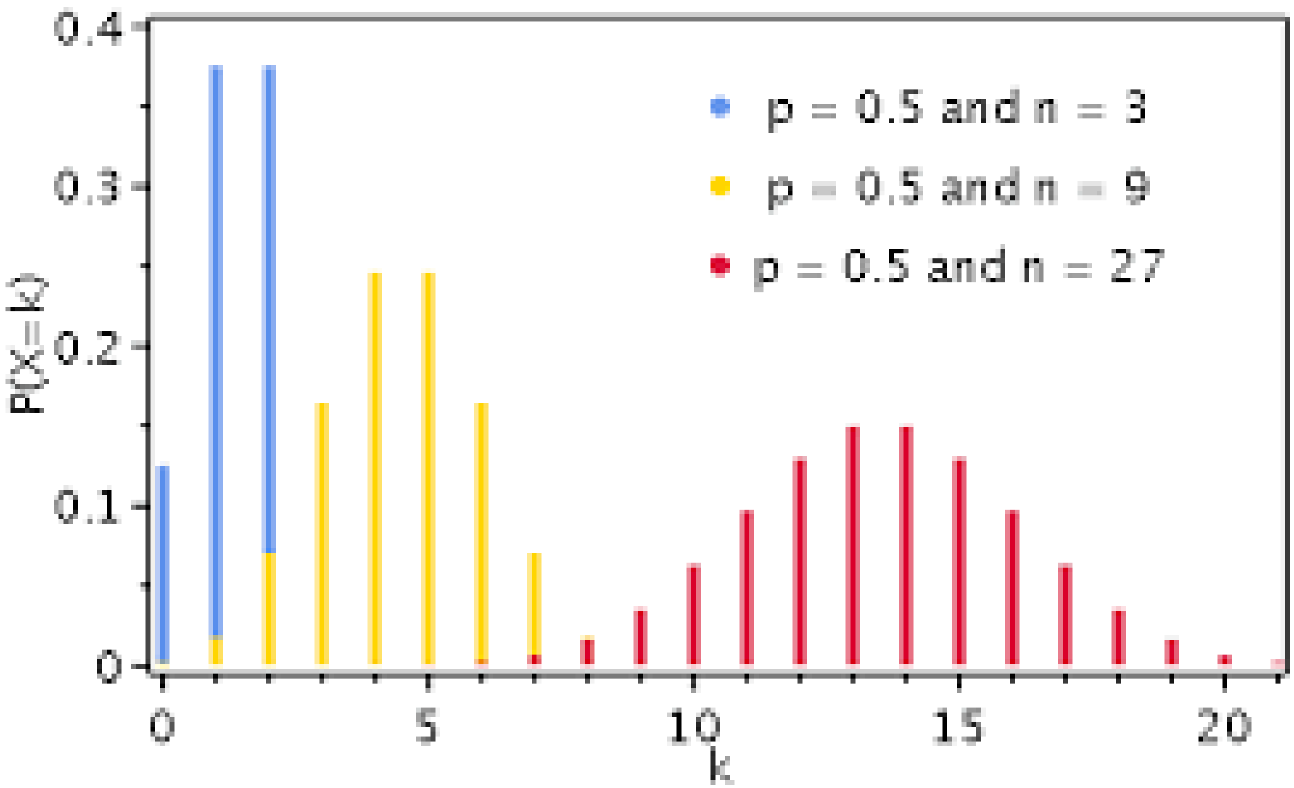
5.5 Normal Approximations to Binomial Distributions

Normal Approximation to a Binomial Distribution

- REVIEW: Binomial Distribution – two outcomes, either ____________________ or _________________________
  - ______________________________________
  - ______________________________________

- If ________________________ and ________________________, then the binomial random variable _________ is approximately _____________________________________, with __________:

- And ____:



p = 0.5 and n = 3
p = 0.5 and n = 9
p = 0.5 and n = 27
P(X=k)
k
0.4
0.3
0.2
0.1
0
0
5
10
15
20

EX:

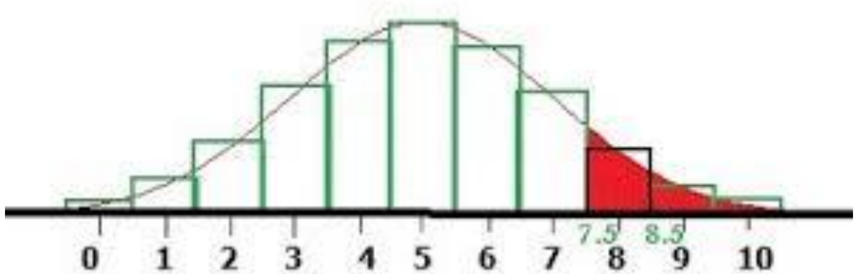
- Two binomial experiments are listed. Determine whether you can use a normal distribution to approximate the distribution of x, the number of people who reply yes. If you can, find the mean and the standard deviation. If you cannot, explain why.
- 1. In a survey of 8 to 18 year old heavy media users in the US, 47% said they get fair to poor grades (C or below). You randomly select forty-five 8 to 18 year old heavy media users and ask them whether they get fair or poor grades.

EX cont.

- 2) In a survey of 8 to 18 year old light media users in the US, 23% said they get fair to poor grades (C or below). You randomly select twenty 8 to 18 year old light media users in the US and ask them whether they get fair or poor grades.

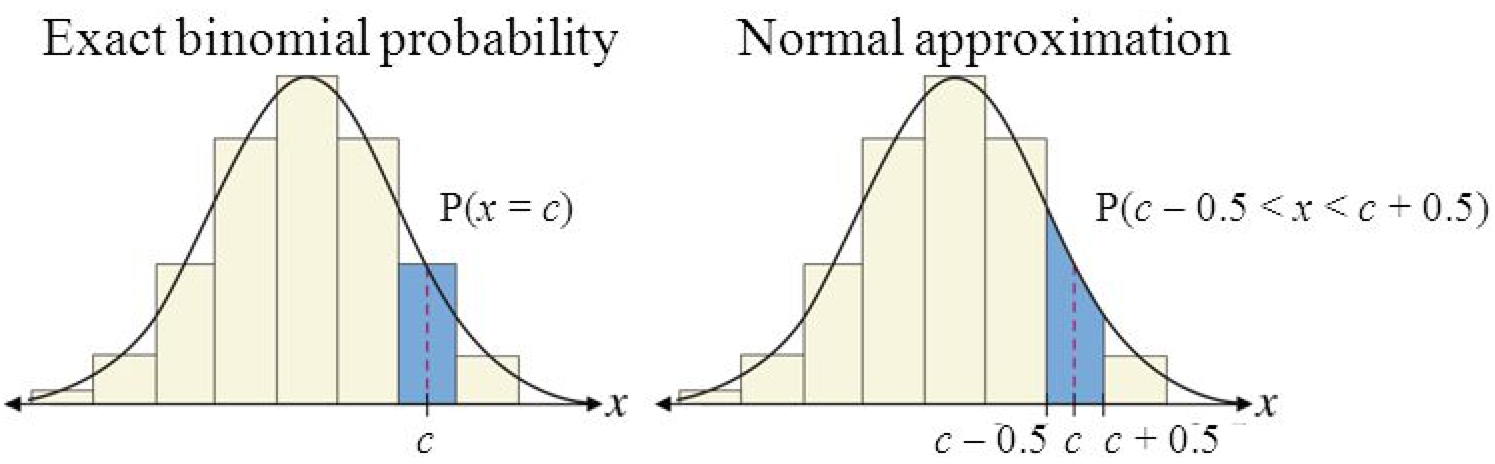
Continuity Correction

- An ______________________________ made when you use a _____________________ normal distribution to _______________________________ a ____________________ __________________________________.
- To include ______________________________ values of _____ in the interval, you need to move _________________ to the _______________ and _______________ of the midpoint.



Correction for Continuity

- When you use a *continuous* normal distribution to approximate a binomial probability, you need to move 0.5 unit to the left and right of the midpoint to include all possible x-values in the interval (**continuity correction**).



EX:

- Use a continuity correction to convert each binomial probability to a normal distribution probability.
- 1) The probability of getting between 270 and 310 successes, inclusive.
- 2) The probability of getting at least 158 successes.
- 3) The probability of getting fewer than 63 successes.
- 4) The probability of getting at most 54 successes.

Guidelines to Using a Normal Distribution to Approximate Binomial Probabilities:

- 1) Verify that a binomial distribution applies.
  - Find _______________________

- 2) Determine whether you can use a _______________________ distribution to approximate ________, the __________________ _____________________________.

  - Is ___________________ and Is ___________________
- 3) Find the ______________________ and the ___________________ _________________________.

Guidelines Cont.

- 4) Apply the ___________________________________. Shade the ______________________________ under the normal curve.

- 5) Find the corresponding _________________________.

- 6)Find the ______________________________ using your _________________________.

EX:

- In a survey of 8 to 18 year old heavy media users in the US, 47% said they get fair to poor grades (C or below). You randomly select forty-five 8 to 18 year old heavy media users and ask them whether they get fair to poor grades. What is the probability that fewer than 20 of them respond yes?

EX:

- Fifty-eight percent of adults say they never wear a helmet when riding a bike. You randomly select 200 adults and ask them whether they wear a helmet. What is the probability that at least 120 adults will say they never wear a helmet when riding a bike?

EX:

- A study of NFL retirees, ages 50 and older, found that 62.4% have arthritis. You randomly select 75 NFL retirees who are at least 50 years old and ask them whether they have arthritis. What is the probability that exactly 48 will say yes?